

Electroweak symmetry breaking in Higgs mechanism with composite operators and solution of naturalness

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Abstract

Introducing a source for a bi-local composite operator motivated by the perturbative expansion in gauge couplings, we calculate its effective potential in the renormalization group of Standard Model with no involvement of technicolor. The potential indicates the breaking of electroweak symmetry below a scale M due to a nonzero vacuum expectation value of neutral component for the $SU(2)$ -doublet operator. At virtualities below a cut off Λ we introduce the local higgs approximation for the effective fields of sources coupled to the composite operators. The value of $\Lambda \approx 600$ GeV is fixed by the measured masses of gauge vector bosons. The exploration of equations for infrared fixed points of calculated Yukawa constants allows us to evaluate the masses of heaviest fermion generation with a good accuracy, so that $m_t(m_t) = 165 \pm 4$ GeV, $m_b(m_b) = 4.18 \pm 0.38$ GeV and $m_\tau(m_\tau) = 1.78 \pm 0.27$ GeV. After a finite renormalization of effective fields for the sources of composite operators, the parameters of effective Higgs field potential are calculated at the scale of matching with the local theory Λ . The fixed point for the Yukawa constant of t quark and the matching condition for the null effective potential at M drive the M value to the GUT scale. The equation for the infrared fixed point of quartic self-action allows us to get estimates for two almost degenerate scalar particles with $m_H = 306 \pm 5$ GeV, while third scalar coupled with the τ lepton is more heavy: $m_{H_\tau} = 552 \pm 9$ GeV. Some phenomenological implications of the offered approach describing the effective scalar field, and a problem on three fermion generations are discussed.

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I. INTRODUCTION

At present, the Standard Model exhibits almost a total success in experimental measurements [1]. The only question being a white spot on its body, is the empirical verification of mechanism for the spontaneous breaking of electroweak symmetry. In this respect, the minimal model involving a single local Higgs field brings a disadvantage: the stability of potential under the quantum loop corrections requires a restriction of quadratic divergency in the self-action by the

introduction of “low” energy cut-off $\Lambda \sim 10^3$ GeV, which is not a natural physical scale standing far away from what can be desirable [2]: the GUT scale, $M_{\text{GUT}} \sim 10^{16}$ GeV [3], or even the Planck mass, $M_{\text{Pl}} \sim 10^{19}$ GeV. The reason for putting the Λ so small, has to originate beyond the Standard Model. Two highways to a “new physics” merit the most popularity. The first one is a technicolor [4] postulating an extra-strong interaction for new technifermions, which form some “QCD-like” condensates, breaking down the electroweak symmetry and giving the masses to the ordinary gauge bosons. Despite some problems with the generation of realistic mass values for the quarks and leptons and suppression of flavor changing neutral currents, the extended technicolor [5] provides quite a clear picture for what happens in the region deeper than 10^3 GeV. However, the most strict objection against such the way is the comparison with the current measurements, which disfavor the technicolor models possessing the calculability [6]. A general consideration of models with the condensation of heavy fermions is reviewed in ref.[7], while a brilliant presentation of both the ideas on the electroweak symmetry breaking with composite operators and techniques as well as results is given in a comprehensive survey by C.T.Hill and E.H.Simmons [8]. However, the condensation, in general, does not provide us with the solution of naturalness. In fact, this approach reformulates the problem as a fine-tuning phenomenon, since the separation of dynamics responsible for the composite operators at a high scale from the low-energy electroweak physics takes place at effective couplings tuned to some critical values. Therefore, we need an additional argumentation in order to address the naturalness in the framework of condensation mechanism with composite operators. We present an idea toward this direction below.

The second way is a supersymmetry [9] reforming the quadratic divergency in the self-action of Higgs field into the logarithmic one, so that it prescribes the scale Λ to be a splitting between the particles of Standard Model and their super-partners. Therefore, the supersymmetry has to be broken in a manner conserving the logarithmic behavior of renormalization, which is an additional challenge to study and a degree of ambiguity. However, the advantage is the stability of Higgs potential, so that Λ certainly is a reasonable scale reflecting the physics in the supersymmetric theory. What remains is the question: why the basic SUSY scale is so “low” in comparison with the GUT scale? Hence, the naturalness is again the problem standing in the higher-quality context.

If the ultraviolet cut off energy in the loop calculations is placed close to the Planck scale (see Fig.1¹), the Standard Model suffers from the inherent inconsistency except a narrow window in the range of possible values of Higgs particle mass: $m_H = 160 \pm 20$ GeV, which does not contradict the value following from the precise measurements of electroweak parameters in the electron-positron annihilation at the Z boson peak.

The reasons for such the inconsistency are the following: At lower masses the vacuum stability is broken, i.e., the quartic coupling constant of scalar field changes its sign [13]. At higher masses the theory enters the strong self-interaction regime, which indicates that the quartic coupling constant becomes infinite (alike the Landau pole) at a scale less than the offered cut off [14]. If the scale of ultraviolet cut off in the SM is much lower than the Planck scale, then the region of higgs masses providing the SM consistency, is more wide. However, such the scales are not natural. A low cut off scale should indicate a new dynamics. While the vacuum instability is an unavoidable physical constraint, the phase of strong higgs

¹The figure originally appeared in ref. [10], and it is taken from ref. [11], while the two-loop consideration recently was done in ref. [12].

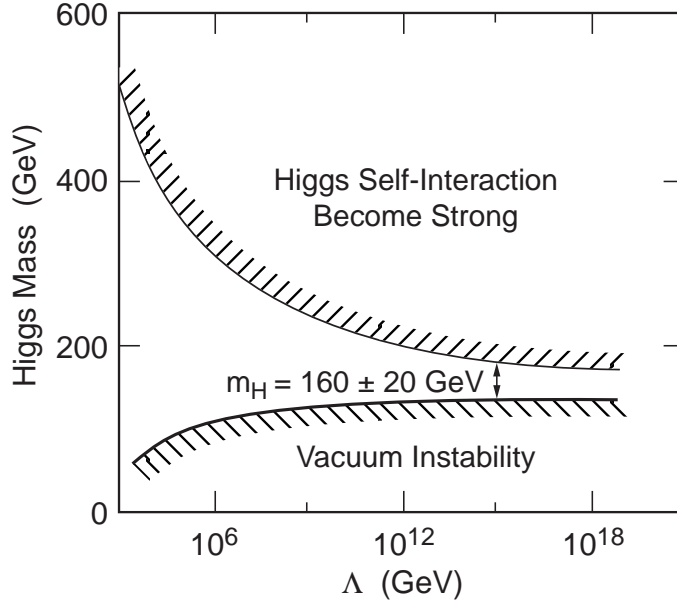


Figure 1: The region of higgs mass constrained by requirements of the SM consistency.

self-interaction could be treated in the framework of the following representation: The scalar higgs can be described in terms of the local field below an ultraviolet cut off Λ placed close to the region of strong regime. At virtualities higher than Λ , the strongly self-coupled higgs is not fundamental and local quantum. The dynamics should be described by means of weakly interacting particles, so that a composite operator with appropriate quantum numbers has to correspond to the higgs in the ‘dual’ limit implying that the effective potential of the composite operator yields a development of vacuum expectation value for the global (independent of space-time point) source of operator. This strong self-interaction regime could be realized with no involvement of an extended underlying theory alike the technicolor dynamics, since some composite operators can develop an appropriate effective potential in the framework of standard electroweak symmetry.

Our assumptions are the followings:

1. We choose a form of composite operators describing the nonlocal phase of higgs in the strong self-interaction regime (SSIR) and suppose the connection of such the operators to the higgses. The suggestion on the nonlocality of higgses allows us to replace the strong self-interaction regime in the theory with the local Higgs fields by the weak self-interaction regime (WSIR) of sources for the composite operators.

2. The interactions of fermions and gauge bosons in the SSIR are given by the dynamics of SM with no local scalar higgses as well as no extensions like a technicolor or so.

3. Concerning the position of scale Λ denoting the infrared cut off in the calculations with the composite operators as well as the ultraviolet cut off in the local theory with the scalar higgs, we put it into the (infrared) fixed point for the Yukawa coupling constants of heaviest fermion in the local theory. the numerical value of Λ is given by the masses of weak-interaction gauge bosons.

4. We consider Yukawa couplings of the only heaviest fermion generation in the SM.

5. In the SSIR we introduce the ultraviolet cut off $M \gg \Lambda$. At M the electroweak symmetry is exactly restored.

6. We match the effective potential of sources for the composite operators with the potential of corresponding local scalar fields at the scale Λ .

The corresponding divisions of virtualities are presented in Fig. 2.

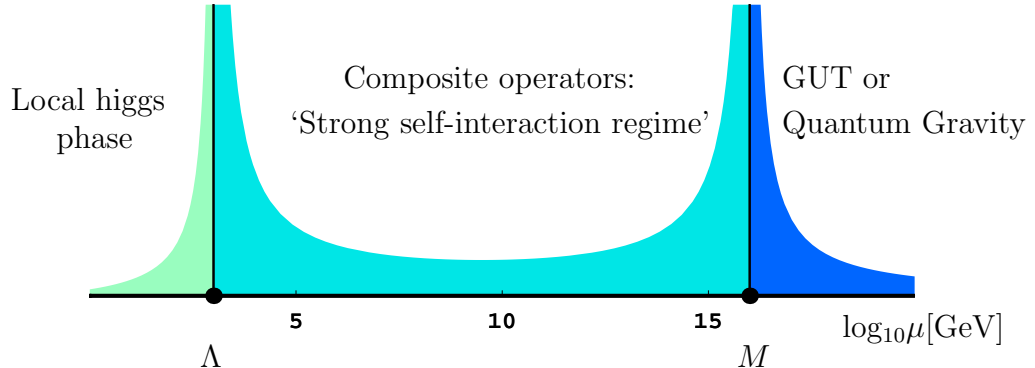


Figure 2: The division of virtualities as accepted in the calculations throughout of this paper.

It is important to stress that the global sources of composite operators develop the Higgs-like potential in the region of $[\Lambda; M]$, so that the corresponding couplings of self-interaction as well as Yukawa constants fall off to zero under the increase of virtuality from Λ to M . Therefore, the dynamics of local interactions is perturbative in the region of $[\Lambda; M]$, while the notion on the “strong self-interaction regime”, strictly speaking, concerns for a theory with the local Higgs field, i.e. we replace $\text{SSIR}|_{\text{local}} \rightarrow \text{WSIR}|_{\text{composite}}$.

Postponing a supersymmetric extension in a time, in this paper we develop a new insight into the breaking of electroweak symmetry by means of exploring the dynamics of SM to calculate an effective potential for a source of bi-local operator with no technicolor interactions. The physical reasoning for the choice of operator under study was hinted in ref.[15]. So, in the second order of perturbation theory we write down the following contribution to the action:

$$iS_{2m} = - \int dxdy \quad \mathbf{T}[\bar{L}_L(x)\not{B}(x)L_L(x) \cdot \bar{L}_R(y)\not{B}(y)L_R(y)] \cdot 4\pi\alpha_Y \cdot \frac{Y_L}{2} \cdot \frac{Y_R}{2}, \quad (1)$$

where we have introduced the notations L_L for the left-handed doublets and L_R for the right-handed singlets, B is the gauge field of weak hypercharge Y , α_Y is its coupling constant. Note, that the gauge field of local $U(1)$ -group is the only one interacting with both the left-handed and right-handed fermions. If we suggest a nontrivial vacuum correlators with the characteristic distance $r \sim 1/v$

$$\begin{aligned} \langle 0 | \mathbf{T}[\not{B}(x)L_L(x) \cdot \bar{L}_R(y)\not{B}(y)] | 0 \rangle &\Rightarrow \frac{\delta(x-y)}{v^4} \langle 0 | \mathbf{T}[\not{B}(x)L_L(x) \cdot \bar{L}_R(x)\not{B}(x)] | 0 \rangle \\ &\sim \delta(x-y) v, \end{aligned} \quad (2)$$

supposing that the scales of expectations for BB and $L_L\bar{L}_R$ are driven by v^2 and v^3 , respec-

tively, then the Dirac masses of fermions are determined by the action²

$$S_{fm} \sim \int dx \bar{L}_L(x) L_R(x) \cdot v \cdot 4\pi\alpha_Y \cdot \frac{Y_L}{2} \cdot \frac{Y_R}{2} + \text{h.c.} \quad (3)$$

In this way we extend the SM action by the initial bi-local bare J -term

$$S_{ib} = \int dx dy N_J \cdot J(x, y) [\bar{L}_R(x) \underbrace{\mathcal{B}^\perp(x) \mathcal{B}^\perp(y)} L_L(y)] - \int dx \phi(x) J(x, x) + \text{h.c.},$$

where $N_J = \pi\alpha_Y \cdot Y_l \cdot Y_R$, and $\underbrace{\cdot \cdot}$ denotes the propagation of transversal U(1)-gauge field $B_\mu^\perp = (g_{\mu\nu} - \partial_\mu \partial_\nu / \partial^2) B^\nu$, which is independent of the longitudinal mode, so that

$$\underbrace{B_\mu^\perp(x) B_\nu^\perp(0)} = -ig_{\mu\nu} \int \frac{d^4 p}{(2\pi)^4} e^{ipx} \frac{1}{p^2}$$

to the leading order of perturbative theory. To the bare order the equation of motion for the bi-local field results in the straightforward substitution of local field ϕ , as it stands in the above consideration for the correlators, developing the vacuum expectation values. After the analysis of divergences in the J -dependent Green functions, the corresponding contra-terms must be added to the action. Then the J -source can be integrated out or renormalized, that results in a Higgs-like action, containing some couplings to fermions as well as a suitable potential to develop the spontaneous breaking of electroweak symmetry.

We stress that there are no other suitable composite operators appearing in the second order of SM gauge symmetry with the quantum numbers relevant to the Higgs interactions providing the generation of fermion masses through the Yukawa-like couplings except the operators described above.

In this paper we calculate the effective potential up to the quartic term for the sources corresponding to the bi-local composite operators of quarks and leptons to the one-loop accuracy of renormalization in the SM. The normalization condition of potential parameters: μ^2 and λ standing in

$$V(J^\dagger, J) = -\mu^2 \cdot J^\dagger J + \lambda \cdot (J^\dagger J)^2,$$

is strictly defined in the SM, since we do not involve some additional interactions. Therefore, both μ^2 and λ for a nonfundamental source must be equal to zero, exactly, i.e. $V = 0$, which, however, can be satisfied at a single scale M because of logarithmic renormalization for couplings, so that

$$\mu^2(M) = 0, \quad \lambda(M) = 0. \quad (4)$$

It is essential that the choice of composite operators is conformed to the effective action of SM in the second order over the gauge couplings. Otherwise, the introduction of arbitrary composite operators with the given properties with respect to the gauge symmetry generally does not imply the imposition of matching condition in (4), which is extremely important, since it removes an uncertainty of the potential due to a finite renormalization of parameters.

²By the way, Eq.(3) implies that in the SM the neutrino is massless since its right-handed component is decoupled, $Y_R = 0$.

Below M , the mass parameter $\mu^2(\Lambda)$ depending on the “infrared cut-off Λ , is positive, and the electroweak symmetry is broken down. So, we suppose that the bi-local representation is valid in the range of virtualities: $[\Lambda; M]$, and below Λ we can explore the local Higgs fields.

As was shown in ref. [16], a variety of composite operators appropriate for the Higgs quantum numbers can be rearranged so that practically arbitrary values of higgs mass or t quark mass could be derived. In other words, in order to get a definite description of higgs sector, one should to suppress contributions by a lot of composite operators except the special ones. this dominance of several composite operators is usually motivated by an extended dynamics beyond the SM, for instance, by the technicolor providing the dominance of some bound channels.

In the present paper, the choice of dominant composite operators is dictated by the SM gauge symmetry, since we isolate the only composite structure in the second order over the gauge couplings, while the appearance of other operators takes place at higher orders, and, hence, their contributions must be suppressed³. Moreover, this motivation on the form of composite operators makes us to add the matching condition of (4), which is a new idea for the composite models, and it is certainly due to the electroweak nature of composite operators.

Thus, for a walker travelling from low scales to higher ones, the whole picture of electroweak symmetry breaking looks as the following:

1. The SM extension with several Higgs fields is the local theory with the ultraviolet cut-off Λ .
2. The parameters of Higgs potential at the scale Λ is matched with the effective potential of bi-local source, calculated in the range $[\Lambda; M]$, so that M denotes the scale, where the potential is exactly zero.

The value of Λ , hence, can be related with the masses of gauge bosons, or the vacuum expectation value (VEV) v_{SM} for the Higgs field in the SM. The value of M with respect to Λ is fixed by two simple requirements: at the matching point Λ the Yukawa constant of t -quark calculated with the composite operators is determined by the condition of infrared fixed point in the local theory [17], while the Yukawa constant is expressed in terms of abelian gauge coupling at the scales of Λ and M due to the consistent matching condition of potential (4). At fixed Λ this supposition makes M to grow to the GUT scale, that implies the solution of naturalness. So, we can read off the third point:

3. The Yukawa constants of heaviest fermions in the local theory have the matching conditions at Λ to the couplings given by the bi-local representation, so that the infrared fixed point for the t -quarks is exactly reached.

The masses of b -quark and τ -lepton can be also calculated after the use of both the definite matching at Λ and infrared fixed points in the RG equations below Λ .

Finally, the potential of Higgs fields at Λ can serve to estimate the masses of neutral scalar particles by means of RG evolution and the infrared fixed point for the quartic vertex λ . The important property of fixed points under consideration is that the Yukawa constants and quartic coupling are given by appropriate combinations of gauge coupling constants.

³This suppression becomes even better at higher virtualities because of the asymptotic freedom of non-abelian interactions, while the abelian charge remains small up to the GUT scale.

Thus, the local theory with the local Higgs fields and the electroweak symmetry breaking can be certainly matched to the effective potential of sources for the bi-local composite operators of quarks and leptons at the scale Λ and to the corresponding Yukawa constants, which are calculable in the region of virtualities $[\Lambda; M]$, so that the fixed point matching of t -quark coupling and the symmetry matching-condition of null effective potential (4) result in M living in the GUT area.

Then we find the following general results:

1. Three bi-local composite operators formed by the fermions of heaviest generation, develop the effective potential of their sources, so that nonzero VEV's break the electroweak symmetry. We treat these dynamics above the scale Λ as the strong self-interaction regime for three independent scalar higgses as equivalent to the weak self-interaction regime for three independent sources of composite operators.

2. The position of matching point $\Lambda = 633$ GeV is fixed by the measured masses of gauge bosons, after the higgs sector is given by three independent scalar fields.

3. At Λ the infrared fixed point condition is satisfied for the Yukawa coupling of t quark, only, while the couplings of b quark and τ lepton evolve to the fixed points at lower scales in agreement with the current data available. The masses of higgses evolve too.

4. Under the item 3, the position of ultraviolet cut off $M \sim 10^{12} - 10^{19}$ GeV with respect to Λ is given by the condition of zero effective potential for the sources of composite operators, that is governed by the renormalization group for the U(1) hypercharge, so that we find the natural hierarchy for M and Λ .

The paper is organized in the following way: Section II is devoted to the definition of sources for the bi-local composite operators and calculation of effective potential to the one-loop accuracy. The masses of gauge bosons and Yukawa constants of fermions are evaluated in Section III at the scale Λ . The exploration of infrared fixed point conditions for the Yukawa constants and quartic Higgs coupling is considered in Section IV. Numerical estimates of masses for the heaviest fermions as well as the Higgs fields are given in Section V. In Section VI we shortly discuss the problem of generations and the vacuum structure. The obtained results and the points of discussion are summarized in Conclusion.

II. SOURCES OF COMPOSITE OPERATORS AND EFFECTIVE POTENTIAL

Let us define the following bare actions for the sources of bi-local operators

$$\begin{aligned}
S_\tau &= \int dxdy N_\tau \cdot J_\tau^\dagger(x, y) [\bar{\tau}_R(x) \underbrace{\not{B}^\perp(x) \not{B}^\perp(y)} \tau_L(y)] + \text{h.c.}, \\
S_t &= \int dxdy N_t \cdot J_t^\dagger(x, y) [\bar{t}_R(x) \cdot n \underbrace{\not{B}^\perp(x) \not{B}^\perp(y)} \bar{n} \cdot t_L(y)] + \text{h.c.}, \\
S_b &= \int dxdy N_b \cdot J_b^\dagger(x, y) [\bar{b}_R(x) \cdot n \underbrace{\not{B}^\perp(x) \not{B}^\perp(y)} \bar{n} \cdot b_L(y)] + \text{h.c.},
\end{aligned} \tag{5}$$

where we have introduced the SU(3)-triplet unit-vector n_i , so that $\bar{n} \cdot n = 1$, and the n -dependent terms in the effective action after the account for the loop-corrections have to be averaged over n_i to restore the explicit invariance under the transformations of SU(3). For

instance, since we generally have $n_i \bar{n}_j = \frac{1}{3} \delta_{ij} + \frac{1}{\sqrt{3}} \lambda_{ij}^a F_a$, we can straightforwardly check that

$$\langle n_i \bar{n}_j \rangle = \frac{1}{3} \delta_{ij}, \quad \langle F_a \rangle = 0, \quad \langle F_a F_b \rangle = \frac{1}{8} \delta_{ab},$$

and so on.

For nonzero Y_L and Y_R , which are under consideration, we can redefine the factors N_J to include the hypercharges into the definition of sources, so that $\tilde{N}_p = \alpha_Y$ and $\tilde{J}_p = \pi \cdot Y_L \cdot Y_R J_p$ for $p = t, b, \tau$, which will not change the final results concerning for the physical quantities: masses and couplings. In what follows we will omit the tildes for the sake of brevity.

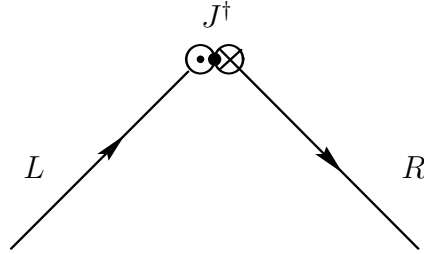


Figure 3: The vertex of global source J^\dagger for the bi-local operator of left-handed and right-handed fermions, where the huge dot denotes the propagation of hypercharge gauge boson.

In the calculations of effective potential we consider the global values of sources independent of local coordinates: $\partial_{x,y} J(x,y) \equiv 0$. The corresponding vertex derived from actions (5) is shown in Fig. 3. For the t -quark it has the form

$$\Gamma_t = i\alpha_Y J^\dagger \bar{t}_R(p) \cdot n \frac{-4i}{p^2} \bar{n} \cdot t_L(p) + \text{h.c.} \quad (6)$$

The diagrams for the calculation of quadratic and quartic terms of effective potential are shown in Figs. 4 and 5, respectively.

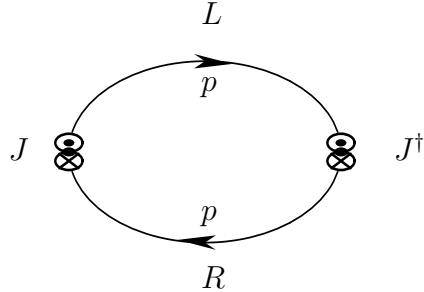


Figure 4: The $J^\dagger J$ -term in the effective potential.

The parameters of potential

$$V(J^\dagger, J) = -\mu^2 \cdot J^\dagger J + \lambda \cdot (J^\dagger J)^2, \quad (7)$$

can be written down in the euclidean space as

$$i\mu_B^2 = -iN_J^2 \int_{\Lambda^2}^{M^2} \frac{d^4p}{(2\pi)^4} \frac{4^2 \text{tr}[P_L \not{p} \not{p}]}{(p^2)^4}, \quad (8)$$

$$-i4\lambda_B = -2iN_J^4 \int_{\Lambda^2}^{M^2} \frac{d^4p}{(2\pi)^4} \frac{4^4 \text{tr}[P_L \not{p} \not{p} \not{p} \not{p}]}{(p^2)^8}, \quad (9)$$

which are independent of the fermion flavor. Here $P_L = \frac{1}{2}(1 - \gamma_5)$ is the projector on the left-handed fermions.

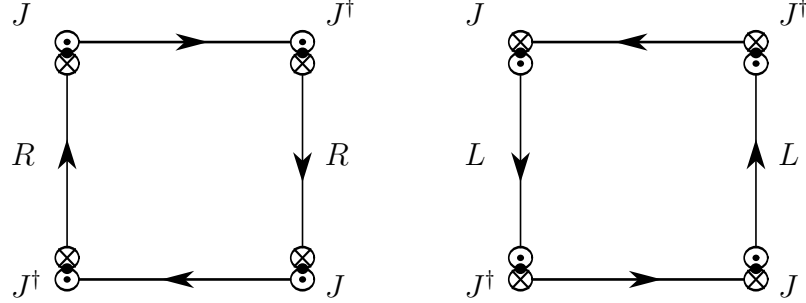


Figure 5: The $(J^\dagger J)^2$ -term in the effective potential.

Supposing $M^2 \gg \Lambda^2$, we find

$$-\mu_B^2 = N_J^2 \frac{2}{\pi^2} \frac{1}{\Lambda^2}, \quad (10)$$

$$\lambda_B = N_J^4 \frac{4}{\pi^2} \frac{1}{\Lambda^8}. \quad (11)$$

As we have already mentioned in the Introduction, the effective potential has to be subtracted, so that at the scale M it equals zero, exactly, since we deal with the source of composite operators not involving some interactions beyond the gauge interactions of SM. Then, we get

$$\mu_R^2(\Lambda) = \frac{2}{\pi^2} \frac{1}{\Lambda^2} \alpha_Y^2(M)(1 - \varkappa^2(\Lambda)), \quad (12)$$

$$\lambda_R(\Lambda) = \frac{4}{\pi^2} \frac{1}{\Lambda^8} \alpha_Y^4(M)(1 - \varkappa^2(\Lambda))^2, \quad (13)$$

where we have introduced the notation for

$$\varkappa(\Lambda) = \frac{\alpha_Y(\Lambda)}{\alpha_Y(M)},$$

with the normalization $\varkappa(M) = 1$. The scale-independent factors $\alpha_Y^{2,4}(M)$ can be removed by the redefinition of sources: $J' = \alpha_Y J$, which we imply below. In addition we introduce $J(\Lambda) = \frac{1}{\Lambda^2} J'$ to obtain more usual notations. Then,

$$\mu^2(\Lambda) = \frac{2}{\pi^2} (1 - \varkappa^2(\Lambda)) \Lambda^2, \quad (14)$$

$$\lambda(\Lambda) = \frac{4}{\pi^2} (1 - \varkappa^2(\Lambda))^2. \quad (15)$$

The vacuum expectation value, VEV, is given by $\langle J^\dagger J \rangle = \frac{\mu^2}{2\lambda}$, so that

$$\langle J^\dagger(\Lambda) J(\Lambda) \rangle = \frac{1}{4} \frac{1}{1 - \kappa^2(\Lambda)} \Lambda^2. \quad (16)$$

Remember, that the potential parameters are the same for all charged heavy fermions: t -quark, b -quark and τ -lepton. The density of vacuum energy is independent of flavor, too,

$$V(\text{vac}) = -\frac{\mu^4}{4\lambda} = -\frac{1}{4\pi^2} \Lambda^4.$$

Then the action represented as the sum of terms over the space-time intervals with $d^4x \sim 1/\Lambda^4$, has the form

$$S(\text{vac}) = - \int d^4x V(\text{vac}) \sim \sum \frac{1}{4\pi^2},$$

and it is independent of Λ .

III. MASSES OF GAUGE BOSONS AND YUKAWA CONSTANTS

The diagrams, which result in the masses of gauge bosons, are shown in Fig. 6, where the permutations over the gauge bosons are implied.

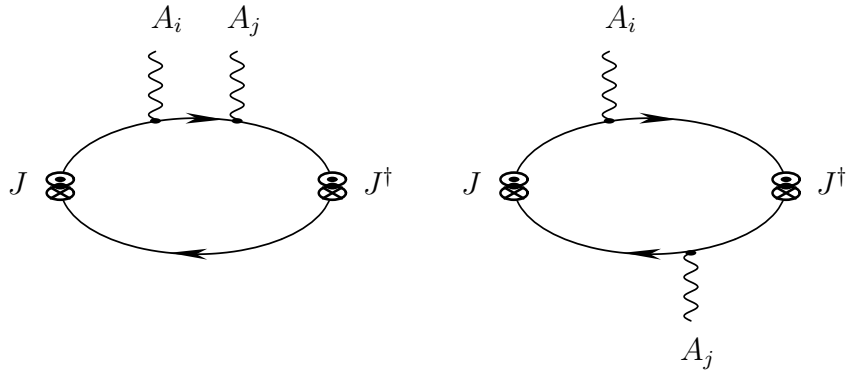


Figure 6: The $(A_i A_j)$ -terms in the effective potential of gauge bosons.

We straightforwardly find that the couplings of gauge bosons are proportional to the differences of their charges, so that

$$m_{12}^2 A_1^\mu A_2^\nu g_{\mu\nu} \sim (Q_1^L - Q_1^R)(Q_2^L - Q_2^R) A_1^\mu A_2^\nu g_{\mu\nu},$$

where $Q^{L,R}$ denote the charges of left-handed and right-handed fermions. This implies that the vector-like gauge bosons, i.e. when $Q^L = Q^R$, remain massless.

For the W - and Z -bosons after the subtraction procedure of $\varkappa^2 \rightarrow (1 - \varkappa^2)$, we find

$$m_W^2 = \frac{4\pi\alpha_2}{2} \sum_p \frac{\Lambda_p^2}{4\pi^2}, \quad (17)$$

$$m_Z^2 = m_W^2 \frac{1}{\cos^2 \theta_W}, \quad (18)$$

where θ_W is the Weinberg angle [18], as usual, and the sum is taken over the heavy flavors $p = t, b, \tau$. As we have seen in the previous section $\Lambda_p = \Lambda$ is independent of flavor, and, hence, we can introduce the Higgs field h_p with the VEV, $\langle h_p \rangle = v$, so that

$$v = \frac{\Lambda}{2\pi}, \quad h_p(v) = \frac{1}{\pi} \sqrt{1 - \varkappa^2(v)} J_p(v).$$

Thus, we get

$$m_W^2 = \frac{4\pi\alpha_2}{2} 3v^2, \quad (19)$$

so that $v_{\text{SM}}^2 = 3v^2 \approx (174 \text{ GeV})^2$, when the potential at the scale Λ has the form⁴

$$V(h_p, h_p^\dagger) = -2\Lambda^2 h_p^\dagger \cdot h_p + (2\pi)^2 (h_p^\dagger \cdot h_p)^2. \quad (20)$$

We check that the quadratic term $-2\Lambda^2$ is exactly given by the one-loop calculation in the local ϕ^4 -theory with $\lambda = (2\pi)^2$ and cut-off Λ .

The masses of fermions at the same scale can be derived from the diagram shown in Fig. 3 by putting the fermion momenta to the given virtuality, $p^2 = \Lambda^2$. Then, after the appropriate subtraction [$\varkappa \rightarrow (1 - \varkappa)$] we get

$$m_p = \lambda_p \cdot v,$$

with

$$\lambda_t(v) = \lambda_b(v) = \frac{4\pi}{3\sqrt{2}} \sqrt{\frac{1 - \varkappa(v)}{1 + \varkappa(v)}}, \quad (21)$$

$$\lambda_\tau(v) = 3\lambda_t(v). \quad (22)$$

Replacing v by v_{SM} , we find $\lambda_p^{\text{SM}} = \lambda_p/\sqrt{3}$.

Thus, we have calculated the masses of gauge bosons, Yukawa constants of heaviest fermions and the parameters of Higgs potential at the scale Λ , which have to be matched with the quantities of local theory valid below Λ .

IV. INFRARED FIXED POINTS

To the moment we have the local theory with three neutral Higgs fields, which are coupled with the appropriate heavy fermions in each sector, with the cut-off Λ , where the Yukawa couplings have to be matched with the values calculated in the effective potential of sources for the composite operators.

⁴There is a possibility to change the convention on the prescription of scale by replacing $\Lambda \rightarrow v$.

The one-loop RG equations for the couplings⁵ have the form [20]

$$\begin{aligned}\frac{d \ln \lambda_t}{d \ln \mu} &= \frac{1}{(4\pi)^2} \left[\frac{9}{2} \lambda_t^2 - 8g_3^2 - \frac{17}{12} g_Y^2 - \frac{9}{4} g_2^2 \right], \\ \frac{d \ln \lambda_b}{d \ln \mu} &= \frac{1}{(4\pi)^2} \left[\frac{9}{2} \lambda_b^2 - 8g_3^2 - \frac{5}{12} g_Y^2 - \frac{9}{4} g_2^2 \right], \\ \frac{d \ln \lambda_\tau}{d \ln \mu} &= \frac{1}{(4\pi)^2} \left[\frac{5}{2} \lambda_\tau^2 - \frac{15}{4} g_Y^2 - \frac{9}{4} g_2^2 \right],\end{aligned}\tag{23}$$

where $g_3^2 = 4\pi\alpha_s$ is the QCD coupling, $g_Y^2 = 4\pi\alpha_Y$ is the hypercharge coupling, and $g_2^2 = 4\pi\alpha_2$ is the SU(2)-group coupling. At “low” virtualities about $v \sim 100$ GeV, the dominant contribution to the β -functions of quark couplings is given by QCD. We suppose that the value of matching point Λ is dictated by the fixed point condition for the t -quark: $\frac{d \ln \lambda_t}{d \ln \mu} = 0$ [17], i.e.

$$\lambda_t^2(v) = \frac{64\pi}{9} \alpha_s(v) + \frac{34\pi}{27} \alpha_Y(v) + 2\pi\alpha_2(v) \approx \frac{64\pi}{9} \alpha_s(v),\tag{24}$$

when the matching gives

$$\lambda_t^2(v) = \frac{8\pi^2}{9} \frac{1 - \varkappa(v)}{1 + \varkappa(v)}.\tag{25}$$

Therefore, we find

$$\varkappa(v) = \frac{1 - \frac{8\alpha_s(v)}{\pi}}{1 + \frac{8\alpha_s(v)}{\pi}}.\tag{26}$$

Due to the contribution by the hypercharge the difference between the RG equations for λ_b and λ_t causes the reach of infrared fixed point for the b -quark at a lower scale than for the t -quark. Indeed, the fixed point condition for the b -quark reads off

$$\frac{9}{2}(\lambda_b^2(\mu) - \lambda_t^2(\mu)) = -g_Y^2(\mu).\tag{27}$$

Making use of matching condition $\lambda_b(v) = \lambda_t(v)$, we can write down

$$\frac{d \ln \lambda_t / \lambda_b}{d \ln \mu} = -\frac{1}{(4\pi)^2} g_Y^2,$$

for small changes, so that

$$\lambda_b(\mu) - \lambda_t(\mu) \approx -\frac{\lambda_t(\mu)}{(4\pi)^2} g_Y^2 \ln \frac{\Lambda}{\mu}.\tag{28}$$

Then, we can derive from (27) and (28) the following estimate of current mass for the b -quark

$$\ln \frac{m_t}{m_b(\hat{v}_b)} = \frac{\pi}{4\alpha_s(m_b(\hat{v}_b))},\tag{29}$$

⁵The corresponding two-loop RG equations are given in ref. [19]. We shortly comment the influence of two-loop corrections below.

where the current mass of t -quark is given by

$$m_t(m_t) = \frac{8}{3} \sqrt{\pi \alpha_s(v)} \cdot v,$$

since the evolution of t -quark mass above the scale v is determined by the running of effective constant, which is negligibly small in the interval $[v, m_t]$, and, hence, $m_t(m_t) \approx m_t(v)$ with quite a high accuracy. The scale of b -quark normalization is given by the following

$$m_b(\hat{v}_b) = \frac{8}{3} \sqrt{\pi \alpha_s(\hat{v}_b)} \cdot \hat{v}_b,$$

and we use the QCD evolution to extract the current mass of b -quark at the scale of its value

$$m_b(m_b) = m_b(\hat{v}_b) \left(\frac{\alpha_s(m_b(m_b))}{\alpha_s(\hat{v}_b)} \right)^{12/25}.$$

Next, we can evaluate the mass of τ -lepton in the same manner. At low energies we modify the RG equation for the τ -coupling, neglecting the four-fermion weak interactions and taking into account the photon contribution. So, we have

$$\frac{d \ln \lambda_\tau}{d \ln \mu} = \frac{1}{(4\pi)^2} \left[\frac{5}{2} \lambda_\tau^2 - 24\pi \alpha_{em} \right], \quad (30)$$

and the infrared fixed point condition reads off

$$\lambda_\tau^2 = \frac{48\pi}{5} \alpha_{em}. \quad (31)$$

The change of λ_τ from the matching value $\lambda_\tau^2 = 9\lambda_t^2 = 64\pi\alpha_s$ can be found in the solution of

$$\frac{d \ln \lambda_\tau}{d \ln \mu} \approx \frac{1}{(4\pi)^2} \frac{5}{2} \cdot 9\lambda_t^2 \approx 40 \frac{\alpha_s}{4\pi}, \quad (32)$$

so that

$$\lambda_\tau(\mu) = \lambda_\tau(v) \cdot \left(\frac{\alpha_s(\mu)}{\alpha_s(v)} \right)^{-\frac{40}{2b_3}}, \quad (33)$$

where $b_3 = 11 - \frac{2}{3}n_f = 9$ at $n_f = 3$. From (31), (33) and $\lambda_t^2 = 64\pi\alpha_s/9$ we deduce the relation

$$\alpha_s(m_\tau) = \alpha_s(v) \cdot \left(\frac{3}{20} \frac{\alpha_{em}(m_\tau)}{\alpha_s(v)} \right)^{-\frac{9}{40}}. \quad (34)$$

Note, that the one-loop evolution to such the large change of scales is quite a rough approximation. To improve the estimate of τ -lepton mass we integrate (32) numerically with the same boundary conditions and extract the value under consideration.

Let us consider the way to estimate the masses of neutral Higgs bosons. The RG equations for the quartic couplings of scalar particles with the heaviest fermions are represented by the following:

$$\frac{d\lambda}{d \ln \mu} = \frac{3}{2\pi^2} \left[\lambda^2 - \frac{a_p}{4} \lambda_p^4 \right], \quad (35)$$

where $a_t = a_b = 1$, $a_\tau = \frac{1}{3}$, and we neglect the contribution given by the electroweak gauge couplings. This approximation is quite reasonable, since at $\Lambda(v)$ the quartic couplings $\lambda(v) = (2\pi)^2$ dominate. For the Higgs fields coupled to the t - and b -quarks, the infrared fixed points coincide with each other to the order under consideration, so that

$$\lambda(\mu_H) \approx \frac{1}{2} \lambda_{t,b}^2(\mu_H) = \frac{32\pi}{9} \alpha_s(\mu_H),$$

which implies that the corresponding masses of scalars are degenerated with a high accuracy. Let us evaluate the scale of reaching the infrared fixed point. The evolution can be approximated at large λ by the equation

$$\frac{1}{\lambda(\mu_H)} = \frac{1}{\lambda(v)} + \frac{3}{2\pi^2} \ln \frac{v}{\mu_H},$$

so that we derive

$$\ln \frac{v}{\mu_H} \approx \frac{3\pi}{16\alpha_s(\mu_H)} - \frac{1}{6}, \quad (36)$$

If we use the RG evolution for the QCD coupling [6]

$$\frac{1}{\alpha_s(\mu_H)} = \frac{1}{\alpha_s(v)} - \frac{b_3}{2\pi} \ln \frac{v}{\mu_H},$$

at $n_f = 5$, we arrive to

$$\ln \frac{v}{\mu_H} \approx \frac{6}{55} \frac{\pi}{\alpha_s(v)}, \quad (37)$$

although the straightforward equation for the scale in (36) can be more accurate numerically.

Thus, following the general relation for the mass of Higgs field,

$$m_H(\mu) = 2\sqrt{\lambda(\mu)} \cdot v,$$

we have the estimates

$$m_H(v) = 4\pi \cdot v, \quad (38)$$

$$m_H(\mu_H) = \frac{8}{3} \sqrt{2\pi\alpha_s(\mu_H)} \cdot v. \quad (39)$$

As for the Higgs field coupled to the τ -lepton, it is quite easily recognize that the corresponding scale μ is much greater than for the scalars coupled with the heaviest quarks, and, hence, its mass is greater than we have considered above. Indeed, we can use the evolution of λ_τ at large scales, where it is driven as $\lambda_\tau = 3\lambda_t$, so that we derive the relation analogous to (36)

$$\ln \frac{v}{\mu_{H_\tau}} \approx \frac{\pi}{16\sqrt{3}\alpha_s(\mu_{H_\tau})} - \frac{1}{6}, \quad (40)$$

and

$$m_{H_\tau}(\mu_{H_\tau}) = 8\sqrt{\frac{2}{\sqrt{3}}\pi\alpha_s(\mu_{H_\tau})} \cdot v.$$

To the moment we are ready to get numerical estimates.

V. NUMERICAL EVALUATION AND THE NATURALNESS

First of all, the VEV's of Higgs fields are directly given by the masses of gauge bosons, so that

$$v = 100.8 \pm 0.1 \text{ GeV},$$

and the cut-off

$$\Lambda = 2\pi v = 633.0 \pm 0.6 \text{ GeV},$$

where we use the experimental data shown in Table 1.

$m_W, \text{ GeV}$	80.41 ± 0.09
α_2^{-1}	29.60 ± 0.04
$m_t, \text{ GeV}$	174 ± 5

Table 1: The experimental data on the electroweak parameters [5,6].

The estimates for the masses of fermions depend on the values of QCD coupling constant. We put the value⁶

$$\alpha_s(m_Z) = 0.122 \pm 0.003,$$

which corresponds to the $\Lambda_{\overline{\text{MS}}}^{(5)} = 255 \pm 45 \text{ MeV}$ in the three-loop approximation for the β -function. We suppose that the threshold values for the changing the number of active quark flavors are equal to $\hat{m}_b = 4.3 \text{ GeV}$ and $\hat{m}_c = 1.3 \text{ GeV}$. The variation of threshold values is not so important in the estimates in contrast to the uncertainty in α_s , which dominates in the error-bars.

Then we can numerically solve the equations in the previous section to find the current masses

$$\begin{aligned} m_t(m_t) &= 165 \pm 1 \text{ GeV}, \\ m_b(m_b) &= 4.18 \pm 0.38 \text{ GeV}, \\ m_\tau(m_\tau) &= 1.78 \pm 0.27 \text{ GeV}. \end{aligned} \tag{41}$$

The one-loop relation of perturbative QCD for the pole mass of quark [6] is given by

$$m^{(p)} = m(m) \left(1 + \frac{4}{3\pi} \alpha_s(m) \right).$$

Then we estimate

$$\begin{aligned} m_t^{(p)} &= 173 \pm 2 \text{ GeV}, \\ m_b^{(p)} &= 4.62 \pm 0.40 \text{ GeV}. \end{aligned}$$

The QED correction to the τ -lepton mass is negligibly small.

⁶The central value is slightly displaced from the “world average” $\alpha_s(m_Z) = 0.119 \pm 0.002$ [21], though it is within the current uncertainty. However, this parameter corresponds to the LEP fit [1] as well as to the recent global fit of structure functions [22].

We see that the t -quark mass is in a good agreement with the direct measurements. The b -quark mass is in the desirable region. It is close to that of estimated in the QCD sum rules [24], where $m_b(m_b) = 4.25 \pm 0.15$ GeV [25], and in the potential approach [26], where $m_b(m_b) = 4.20 \pm 0.06$ GeV. It is worth to note that the pole mass is not the value, which has a good convergency in the OPE approach (see references in [25, 26]), so we present it to the first order for the sake of reference. However, we stress also that the deviations from the central values are caused by the uncertainties in the α_s running.

The infrared fixed masses of neutral scalars, coupled with the t - and b -quarks and the τ -lepton, equal

$$m_H = 306 \pm 5 \text{ GeV}, \quad m_{H_\tau} = 552 \pm 9 \text{ GeV}, \quad (42)$$

which can be compared with the global fit of SM at LEP yielding $m_H = 76^{+85}_{-47}$ GeV [1]. The central value of this fit was recently excluded by the direct searches at modern LEP energies, where the constraint was obtained $m_H > 95$ GeV [1, 6]. We expect, however, that many-doublet models of Higgs sector have a different connection to the LEP data. Indeed, the fit of SM with the single Higgs particle yields the value for the logarithm $l_H = \log_{10} m_H^{\text{SM}} [\text{GeV}] = 1.88^{+0.33}_{-0.41}$, whereas this correction basically contributes into the observed quantities due to the coupling to the massive gauge bosons. Then, we can write down the following approximation for this value in the model under consideration:

$$l_H = \frac{1}{3} \sum_p \kappa_p \log_{10} m_{H_p} [\text{GeV}],$$

where the factor $\frac{1}{3}$ represents the fraction of scalar coupling in the squares of gauge boson masses, respectively for $p = t, b, \tau$, and κ_p stands for the possible formfactors at high virtualities of the order of masses of Higgs fields. To test, we put the simple approximation

$$\kappa_p \approx \frac{1}{1 + \frac{m_{H_p}^2}{\Lambda^2}},$$

which results in $\kappa_t = \kappa_b$ close to unit, and $\kappa_\tau \sim \frac{1}{4}$, so that the value under consideration is equal to

$$l_H \approx 1.86,$$

that is optimistically close to what was observed at LEP. So, the values in (42) are not in contradiction with the current data.

Next, since we deal with the strongly coupled version of Higgs sector (remember, that $m_H(v) \approx 1267$ GeV), we need more careful consideration of effective potential to take into account the higher dimensional operators, representing the multi-higgs couplings. So, we keep (42) as soft estimates of masses for the Higgs fields, which implies that the decays into the massive gauge bosons are the dominant modes for these scalar particles.

Finally, we evaluate the scale M , where the electroweak symmetry has to be exactly restored. The value of $\varkappa(v)$ is equal to

$$\varkappa(v) = \frac{\alpha_Y(v)}{\alpha_Y(M)} = 0.532 \pm 0.005,$$

which implies $\alpha_1^{-1}(M) \approx 32$. The implication of \varkappa for M depends on the running of $\alpha_Y = \frac{3}{5}\alpha_1$ [6]:

$$\frac{1}{\alpha_1(M)} = \frac{1}{\alpha_1(v)} + \frac{b_1}{2\pi} \ln \frac{M}{v},$$

where b_1 is model-dependent. So, in the SM $b_1 = -\frac{4}{3}n_g - \frac{1}{10}n_h$ with $n_g = 3$ being the number of fermion generations, n_h is the number of Higgs doublets, we obtain⁷

$$M_{\text{SM}} \approx 2.5 \cdot 10^{19} \text{ GeV},$$

when in the SUSY extension $b_1 = -2n_g - \frac{3}{10}n_h$, so that

$$M_{\text{SUSY}} \approx 7 \cdot 10^{12} \text{ GeV}.$$

Hence, we obtain the broad constraints

$$M = 7 \cdot 10^{12} - 2.5 \cdot 10^{19} \text{ GeV},$$

and the value strongly depends on the set of fields in the region above the cut-off Λ . At present, we cannot strictly draw a conclusion on a preferable point. However, we can state that the offered mechanism for the breakdown of the electroweak symmetry solves the problem of naturalness, since the observed “low” scale of gauge boson masses is reasonably related to the “high” scale of GUT or even Planck mass.

Finally, we comment on possible uncertainties of numerical estimates and a role of two-loop corrections. First, we analyze the subleading terms in eq.(24). The gauge charges neglected in the fixed point condition of (24) result in the displacement of t quark mass by a value about 4 GeV, if we do not change the normalization of QCD coupling constant. In this way, we note that under the account of gauge charge corrections in (24) the same central value of t quark mass, i.e. 165 GeV, is reproduced at $\alpha_s(m_Z^2) = 0.118$, which coincides with the Particle Data Group “world-average”. Next, the two-loop corrections in the RG equations for the Yukawa couplings [19] as applied to the t quark lead to an additional displacement of fixed point value. However, in this case we have to take into account the one-loop correction to the relation between the current mass and the pole mass of t quark due to the Higgs sector, that results in the following additive renormalization of $m_t(m_t)$ [23]

$$\frac{\delta m_t(m_t)}{m_t(m_t)} = -\frac{1}{16\pi^2} \frac{9}{2} \frac{m_t^2}{v_{\text{SM}}^2},$$

at the higgs mass $m_H \approx 2m_t$. The above correction compensates the displacement due to the two-loop modification of infrared fixed-point condition for the t quark.

Second, we study the two-loop corrections to the fixed point condition for the b quark. The corresponding modification of (27) reads off

$$\frac{9}{2}(\lambda_b^2(\mu) - \lambda_t^2(\mu)) \approx -g_Y^2(\mu) \left(1 - \frac{1}{16\pi^2} \frac{4}{3} \lambda_t^2(\mu)\right) + O(g^4), \quad (43)$$

that results in the appropriate correction in (28), *viz.*, in the small change of slope in front of log about 2%. The solution of equations for the running b quark mass under the variations

⁷Numerically, we put $\alpha_1^{-1} = 58.6$ for the order-of-magnitude estimate.

caused by the introduction of two-loop corrections and the uncertainty in the coupling constant of QCD is shown in Fig. 7. We can straightforwardly see that the variation of slope in the RG equation for the Yukawa constant of b quark due to the two-loop corrections results in uncertainties, which are much less than the variation of b quark mass caused by the uncertainties in the running coupling constant of QCD at moderate virtualities about the b quark mass. Therefore, the dominant origin of uncertainty for the b quark mass is the normalization of α_s . The same conclusion can be drawn for the mass of τ lepton. Therefore, the uncertainty in the estimates of masses for the b quark and the τ lepton is not essentially changed by the introduction of two-loop corrections, while the value of t quark mass depends on the normalization of α_s as well as the two-loop corrections combined, so that the uncertainty in the current mass can reach 4 GeV in m_t .

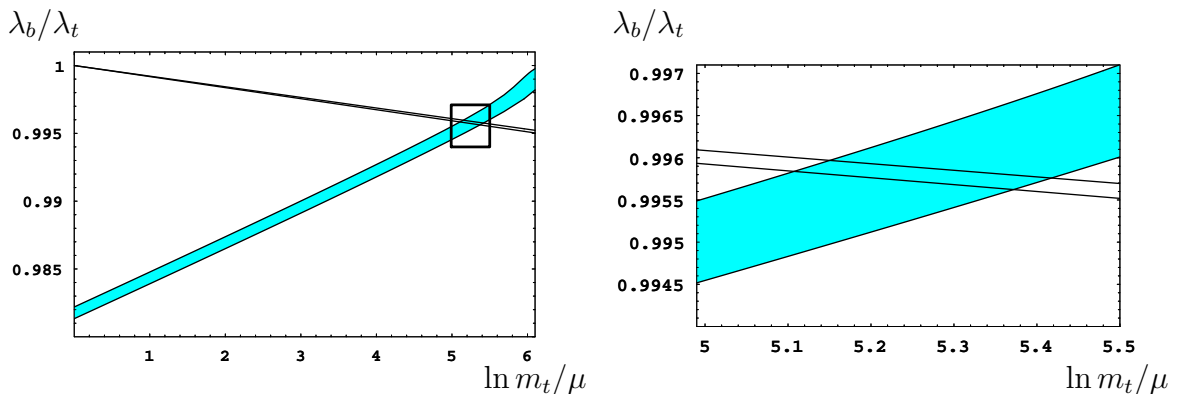


Figure 7: The variation of RG solution for the b quark Yukawa constant under the introduction of two-loop corrections with respect to the one-loop result (solid lines) and the uncertainty caused by the change in the normalization of α_s with $\Lambda_{\overline{\text{MS}}}^{(5)} = 200$ MeV and $\Lambda_{\overline{\text{MS}}}^{(5)} = 280$ MeV (the band). At the right figure we scale the square region marked in the left picture.

As for the estimates of masses for the scalar fields, we emphasize that they give preliminary results, and further investigations are in progress, since we should, first, sum up subleading terms with higher powers of the higgs field squared in the effective potential and, second, consider complete RG equations for the quartic self-coupling, including suppressed terms. Nevertheless, our preliminary estimates show that the scalar fields should be significantly heavy.

VI. GENERATIONS, THE NUMBER OF HIGGS FIELDS AND VACUUM

In the previous sections we have introduced three independent global sources for the bi-local operators composed by the fermions of the heaviest generation, i.e., t quark, b quark and τ lepton. In the SSIR, these sources acquire the effective potentials providing the spontaneous breaking of electroweak symmetry. Below the scale Λ we assume the connection of such the potentials with the potentials of local Higgs fields. Thus, we suppose the introduction of three independent local Higgs doublets at the low energies. Therefore, we suggest the condensation of sources related with the heaviest generation only.

We have found the ‘democratic’ form for the potentials of independent sources in the SSIR. All three potentials have the same values of quadratic and quartic couplings, while we suggest evidently broken ‘democracy’ for the fermion generations, since we do not introduce the condensation of sources for the composite operators built of junior fermions.

In this section we describe a possible development on the problem of fermion generations and the structure of vacuum in the Higgs sector.

So, let us introduce the notation of normalized VEV’s for the global sources connected with the Higgs fields as follows:

$$\chi_p = h_p/v, \quad p = \tau, t, b,$$

and the corresponding vacua $|0_p\rangle$, so that

$$\langle 0_p | \chi_{p'} | 0_{p''} \rangle = \delta_{pp'} \delta_{p'p''}. \quad (44)$$

Then we easily find that the mass terms

$$\begin{aligned} \mathcal{L}_Y^\tau &\sim \bar{\tau}_R \tau_L \cdot \chi_\tau + \text{h.c.}, \\ \mathcal{L}_Y^t &\sim \bar{t}_R t_L \cdot \chi_t + \text{h.c.}, \\ \mathcal{L}_Y^b &\sim \bar{b}_R b_L \cdot \chi_b + \text{h.c.}, \end{aligned} \quad (45)$$

could be represented by means of fields

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \cdot \begin{pmatrix} \chi_\tau \\ \chi_t \\ \chi_b \end{pmatrix} = U \cdot \begin{pmatrix} \chi_\tau \\ \chi_t \\ \chi_b \end{pmatrix}, \quad (46)$$

as follows

$$\begin{aligned} \mathcal{L}_Y^\tau &\sim \bar{\tau}_R \tau_L \cdot (\varphi_1 + \varphi_2 + \varphi_3) + \text{h.c.}, \\ \mathcal{L}_Y^t &\sim \bar{t}_R t_L \cdot (\varphi_1 + \omega^2 \varphi_2 + \omega \varphi_3) + \text{h.c.}, \\ \mathcal{L}_Y^b &\sim \bar{b}_R b_L \cdot (\varphi_1 + \omega \varphi_2 + \omega^2 \varphi_3) + \text{h.c.}, \end{aligned} \quad (47)$$

where we omit the Yukawa couplings and use the matrix U defined in terms of $\omega = \exp(i\frac{2\pi}{3})$.

Such the transformation in (46) relates the ‘heavy’ basis of χ_p with the ‘democratic’ basis of φ_i . So, the definition (46) can be equivalently changed by permutations of $\chi_\tau \leftrightarrow \chi_t \leftrightarrow \chi_b$ or permutations of columns in the matrix U . Such the permutations correspond to the finite cyclic group \mathbb{Z}_3 with the basis ω , so that complex phases of φ_i are given by $e^{iq\frac{2\pi}{3}}$ with the charges $q = (0, -1, 1)$ of $(\varphi_1, \varphi_2, \varphi_3)$.

Further, we can note that the vacuum fields have simple connections as follows:

$$\begin{aligned} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} &\Rightarrow \langle 0_\tau | \chi_1 | 0_\tau \rangle = \langle 0_\tau | \begin{pmatrix} \chi_\tau \\ \chi_t \\ \chi_b \end{pmatrix} | 0_\tau \rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \\ \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ \omega \\ \omega^2 \end{pmatrix} &\Rightarrow \langle 0_t | \chi_2 | 0_t \rangle = \langle 0_t | \begin{pmatrix} \chi_\tau \\ \chi_t \\ \chi_b \end{pmatrix} | 0_t \rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \\ \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ \omega^2 \\ \omega \end{pmatrix} &\Rightarrow \langle 0_b | \chi_3 | 0_b \rangle = \langle 0_b | \begin{pmatrix} \chi_\tau \\ \chi_t \\ \chi_b \end{pmatrix} | 0_b \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \end{aligned} \quad (48)$$

where the conditions of normalization (44) are reproduced.

Let us *postulate the extended definition of the vacuum*

$$|vac\rangle = |0_\tau\rangle \otimes |0_t\rangle \otimes |0_b\rangle, \quad (49)$$

which implies the \mathbb{Z}_3 symmetry of the vacuum. Then, the couplings introduced in (47) are extended to three generations of fermions, whereas the only generation of τ , t and b is heavy, while two junior generations are massless.

The vacuum definition (49) could be treated as the following assumption:

The number of generations equals the number of charged flavors in the generation as well as the number of Higgs fields in the local phase.

Thus, we postulate the \mathbb{Z}_3 symmetry of the vacuum as the fundamental dynamical principal of the theory. Moreover, we suggest that this symmetry of the vacuum is exact, so that it is conserved under radiative corrections to the Yukawa constants of fermions.

A realistic description of generations, i.e., a model with nonzero masses of junior fermions is not the problem under the current consideration, and it is beyond the scope of this work. Nevertheless, we add two notes.

First, a general structure of Yukawa interactions with the \mathbb{Z}_3 symmetry of the vacuum has the form

$$\begin{aligned} \mathcal{L}_Y^\tau &\sim \bar{\tau}_R \tau_L \cdot (g_1^\tau \varphi_1 + g_2^\tau \varphi_2 + g_3^\tau \varphi_3) + \text{h.c.}, \\ \mathcal{L}_Y^t &\sim \bar{t}_R t_L \cdot (g_1^t \varphi_1 + g_2^t \omega^2 \varphi_2 + g_3^t \omega \varphi_3) + \text{h.c.}, \\ \mathcal{L}_Y^b &\sim \bar{b}_R b_L \cdot (g_1^b \varphi_1 + g_2^b \omega \varphi_2 + g_3^b \omega^2 \varphi_3) + \text{h.c.}, \end{aligned} \quad (50)$$

where the constants g_i can be restricted by the following conditions: $g_{2,3}$ are real, while g_1 can be complex. So, the symmetric point

$$g_1 = g_2 = g_3 = 1,$$

restores the hierarchy of single heavy generation and two massless generations. The development of *ansatz* (50) with the realistic values of parameters consistent with the current data on the quark masses and mixing CKM matrix of charged quark currents was given in Ref. [27].

Second, the same form of mass matrix following from (50) could result in the leading order symmetry in the sector of neutrinos. Indeed, if we put

$$g_1 = e^{i\frac{2\pi}{3}}, \quad |g_1| = g_2 = g_3 = 1,$$

then we get the completely degenerate neutrinos, while small deviations in the g_1 phase and absolute values of g_i will result in small differences of neutrino masses squared as observed in the neutrino oscillations [28].

VII. CONCLUSION

In this work we have argued that there are three starting points for the presented consideration of Higgs sector of electroweak theory. These motivations are the following:

First, as well known, in the Standard Model the Yukawa coupling constant of t quark obtained from the measurement of t quark mass, is close to its value in the infrared fixed point

derived from the renormalization group equation. If the Higgs sector is extended to three scalar fields separately coupled to three heaviest charged fermions, i.e., the t , b quarks and τ lepton, with the same vacuum expectation values of Higgs fields, then the Yukawa coupling constant of t quark is **exactly** posed to the infrared fixed point. The challenge is whether this coincidence is accidental or not. We treat this fact as the fundamental feature of dynamics determining the development of masses. Moreover, the problem acquires an additional insight because the only fermion generation is heavy, while two junior generations are approximately massless. These features can be attributed by introducing the fundamental \mathbb{Z}_3 symmetry of the vacuum, so that this symmetry is conserved under the radiative loop corrections responsible for the development of nonzero masses of junior generations.

Second, the strong self-interaction regime in the Higgs sector of Standard Model at large virtualities can be treated as the indication of nonlocality, i.e., the compositeness of operators relevant to the electroweak symmetry breaking. We have introduced a separation of virtuality regions: the local Higgs phase in the range of $[0; \Lambda]$, the nonlocal strong self-interaction regime in the range $[\Lambda; M]$, and the symmetric phase above M . We have determined a form of composite operators and their connection to the local phase by considering the second order of effective action in the SM.

Third, the development of effective potential for the global sources of composite operators from the point of symmetric phase $M \sim 10^{12} - 10^{19}$ GeV is stopped in the infrared fixed point $\Lambda \approx 633$ GeV for the Yukawa coupling constant of t quark. If the dynamics of evolution is given by the same electroweak group, then the large logarithm of $\ln M/\Lambda$ is close to the value appearing in the calculation of GUT scale. So, since the breaking of electroweak symmetry and the fermion mass generation involve the composite operators with both left and right handed fermions, the gauge interaction of $U(1)$ group determines the evolution of parameters of the effective potential in the strong self-interaction regime. Then the logarithm of $\ln M/\Lambda$ in the coupling g_1 has the value depending on the set of fundamental fields above the scale Λ . Anyway, M should be close to the GUT scale, and this fact implies the solution of problem on the naturalness.

Then, following the above motivations, we have evaluated the basic parameters of the model. We have calculated the effective potential for the sources of composite operators, responsible for the breaking down the electroweak symmetry and generation of masses for the gauge bosons and heaviest fermions. The corresponding couplings serve as the matching values for the quadratic and quartic constants in the potential of local Higgs fields as well as the Yukawa interactions at the scale Λ , which is the ultraviolet cut-off for the local theory and the low boundary of $[\Lambda; M]$ -range for the effective potential of sources coupled with the bi-local composite operators of quarks and leptons. At M the local gauge symmetry is restored, so that the effective potential is exactly equal to zero.

Posing the matching of Yukawa constant for the t -quark to the infrared fixed point at the scale Λ , related to the gauge boson masses, we have found the null-potential value M in the range of GUT park, which indicates the solution of naturalness. The exploration of fixed points has resulted in the following current masses of heaviest fermions: $m_t(m_t) = 165 \pm 4$ GeV, $m_b(m_b) = 4.18 \pm 0.38$ GeV and $m_\tau(m_\tau) = 1.78 \pm 0.27$ GeV. Two degenerated neutral Higgs fields have the infrared fixed mass $m_H = 306 \pm 5$ GeV, and the third scalar has the mass $m_{H_\tau} = 552 \pm 9$ GeV. So, the estimates do not contradict with the current constraints, coming from the experimental data.

Some questions need for an additional consideration. To the moment, discussing no possible ways to study, we focus on the directions requiring a progress.

1. What is a picture for the generation of Yukawa constants, responsible for the masses of “junior” fermions?

As we have supposed in the paper, three sectors of Higgs fields are coupled to the appropriate heavy fermions, so that we need speculations based on a symmetry causing the junior generations to be massless to the leading order.

2. What are the constraints on the model parameters as follows from the current data on the flavor changing neutral currents and precision measurements at LEP?

So, we expect that this point is not able to bring serious objections against the model, since we do not involve any interactions distinct from the gauge ones, composing the SM.

3. The most constructive question is a supersymmetric extension of mechanism under consideration. Can SUSY provide new features or yield masses of super-partners?

To our opinion, the SUSY extension is more complicated, since there are many different relations between the mixtures of various sparticles, which all are expected to be essentially massive ($\tilde{m} \sim \Lambda$) in contrast to the SM, wherein the junior generations are decoupled from the Higgs fields to the leading order.

4. A simple application, we think, is an insertion of the model into the TeV-scale Kaluza-Klein ideology, being under intensive progress now [29].

So, $\varkappa(v)$ transforms its logarithmic behavior to the power dependence on the scale. Then, the M returns to a value not far away from the matching point $\Lambda \sim 1$ TeV, as it should be in the KK approach.

Thus, we have offered the model of electroweak symmetry breaking, which provides a positive connection to the naturalness as well as needs some deeper studies under progress.

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